

Optimal control of ratchets without spatial asymmetry

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2007 J. Phys. A: Math. Theor. 40 F413

(<http://iopscience.iop.org/1751-8121/40/22/F01>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.109

The article was downloaded on 03/06/2010 at 05:12

Please note that [terms and conditions apply](#).

FAST TRACK COMMUNICATION

Optimal control of ratchets without spatial asymmetry**Ricardo Chacón**

Departamento de Física Aplicada, Escuela de Ingenierías Industriales, Universidad de Extremadura, Apartado Postal 382, E-06071 Badajoz, Spain

Received 1 February 2007, in final form 18 April 2007

Published 14 May 2007

Online at stacks.iop.org/JPhysA/40/F413**Abstract**

This work presents the following conjecture: to optimally enhance directed transport by symmetry breaking of temporal forces there exists a particular force waveform which allows us to deduce simple scaling laws from a quantitative interpretation of Curie's principle. These scaling laws explain in a general setting previous results for a great diversity of systems subjected to a standard biharmonic force and provide a quantitative criterion to optimize the application of the ratchet effect induced by symmetry breaking of temporal forces. Mathematical arguments justifying this conjecture are discussed.

PACS number: 05.60.–k

Understanding the ratchet effect [1–4] induced by symmetry breaking of temporal forces is a fundamental issue that has remained unresolved for decades. While the dependence of the directed transport on each of the ratchet-controlling parameters has been individually investigated, there is still no general criterion to apply to the whole set of these parameters to optimally control directed transport in general systems without a ratchet potential [5–25]. Without regard for any specific physical context, let us consider a quite general system described by a perturbed Hamiltonian and subjected to a T -periodic zero-mean ac force $f(t)$ where a ratchet effect is induced by solely violating temporal symmetries. A popular choice would be the simple case of a biharmonic force, $f_{h_1, h_2}(t) = \epsilon_1 \text{har}_1(\omega t + \varphi_1) + \epsilon_2 \text{har}_2(2\omega t + \varphi_2)$, where $\text{har}_{1,2}$ represents indistinctly sin or cos. Clearly, the aforementioned symmetries are solely the shift symmetry of the force ($f(t) = -f(t+T/2)$, $T \equiv 2\pi/\omega$) and the time-reversal symmetry of the system's dynamic equations. Of course, the breaking of the latter symmetry implies the breaking of some time-reversal symmetry of the force ($f(-t) = \pm f(t)$) in some general case, but not in all cases [19]. The analysis of the breaking of these two fundamental symmetries allows us to find the regions of the parameter space $(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2)$, $\epsilon_1 + \epsilon_2 = \text{const}$, where the ratchet effect is optimal in the sense that the average of relevant observables (such as velocity and current, hereafter referred to as $\langle V \rangle$) is maximal, the remaining parameters being held constant. In this paper, I conjecture that such regions are those where the *effective* degree of symmetry breaking is *maximal*. The approach arises from the observation that Curie's principle [26] implies that a broken symmetry is a structurally stable situation [4]. At this point, a *quantitative* measure of the degree of symmetry breaking (DSB) is introduced, on

which the strength of directed transport must depend. This quantitative relationship between cause (symmetry breaking) and effect (directed transport) is hereafter referred to as the DSB mechanism. Also, this quantitative relationship is expected to exhibit a dependence on the symmetry-breaking parameters which is general if and only if the symmetry breaking takes place solely in the driving force, i.e., in the external agent which is simultaneously the transport-inducing force and the ratchet-inducing force. Since the ratchet effect can occur at any spatio-temporal scale, such a quantitative measure of the DSB must be independent of the force's amplitude and period. I consistently define the DSB of the symmetries of the force $f(t)$ by the expressions

$$\begin{aligned} D_s(f) &\equiv \left\langle \frac{-f(t+T/2)}{f(t)} \right\rangle_T \equiv \frac{1}{T} \int_0^T \frac{-f(t+T/2)}{f(t)} dt, \\ D_+(f) &\equiv \left\langle \frac{f(-t)}{f(t)} \right\rangle_T \equiv \frac{1}{T} \int_0^T \frac{f(-t)}{f(t)} dt, \\ D_-(f) &\equiv -D_+(f), \end{aligned} \quad (1)$$

where increasing deviation of $D_{s,+,-}(f)$ from 1 (unbroken symmetry, respectively) indicates an increase in the DSB. But the effectiveness of any periodic zero-mean force at producing transport diminishes as the transmitted impulse over a half-period is decreased while its amplitude and period are held constant. In general, this means that optimal enhancement of the ratchet effect is achieved when maximal *effective* (critical) symmetry breaking occurs, which is in turn a consequence of two reshaping-induced competing effects: the increase of the degree of breaking of the force's symmetries and the decrease of the transmitted impulse over a half-period, thus implying the existence of a *particular* force waveform which optimally enhances the ratchet effect. Thus, for the biharmonic force

$$f_{\cos,\cos}(t) = \epsilon_1 \cos(\omega t + \varphi_1) + \epsilon_2 \cos(2\omega t + \varphi_2),$$

equation (1) can be put into the form

$$\begin{aligned} D_s(f_{\cos,\cos}) &= 1 - \frac{a}{\pi} \int_0^{2\pi} \frac{\cos(2\tau + \varphi_{\text{eff}})}{P(\tau; a, \varphi_{\text{eff}})} d\tau, \\ D_+(f_{\cos,\cos}) &= -D_-(f_{\cos,\cos}) = 1 + \frac{a}{\pi} \int_0^{2\pi} \frac{\sin(2\tau) \sin \varphi_{\text{eff}}}{P(\tau; a, \varphi_{\text{eff}})} d\tau, \end{aligned}$$

where $a \equiv \epsilon_2/\epsilon_1$, $\tau \equiv \omega t + \varphi_1$, $\varphi_{\text{eff}} \equiv \varphi_2 - 2\varphi_1$, $P(\tau; a, \varphi_{\text{eff}}) \equiv \cos \tau + a \cos(2\tau + \varphi_{\text{eff}})$. The quantity φ_{eff} is hereafter referred to as the *effective phase*. These integrals diverge at the zeros of the quartic polynomial

$$4a^2 x^4 + 4a \cos \varphi_{\text{eff}} x^3 + (1 - 4a^2) x^2 - 2a \cos \varphi_{\text{eff}} x + a^2 \cos^2 \varphi_{\text{eff}} = 0,$$

where $x \equiv \cos \tau$. After solving this algebraic equation for x , one obtains that the three integrals diverge when $a \geq 1/2$, $\varphi_{\text{eff}} = \{\pi/2, 3\pi/2\}$, and thus the DSB is maximal at these parameter values for the three symmetries of the force (see figure 1(d)). For these values of the effective phase, one finds that the transmitted impulse over a half-period is maximal at $a = 1/2$ while the biharmonic force's amplitude is held constant. This means that maximal *effective* symmetry breaking occurs at $a = 1/2$, $\varphi_{\text{eff}} = \{\pi/2, 3\pi/2\}$. A similar analysis of the remaining three versions of the biharmonic force yields the results summarized in table 1 (second column), which are again the same for the three symmetries in each case. Note that one could equivalently define a measure of the DSB by taking the time average of the inverse quantities $-f(t)/f(t+T/2)$, $\pm f(t)/f(-t)$: one finds that the corresponding measure (1) exhibits the same qualitative behaviour as a function of the symmetry-breaking parameters, and exactly the same optimal values of these parameters are found to yield a maximal effective

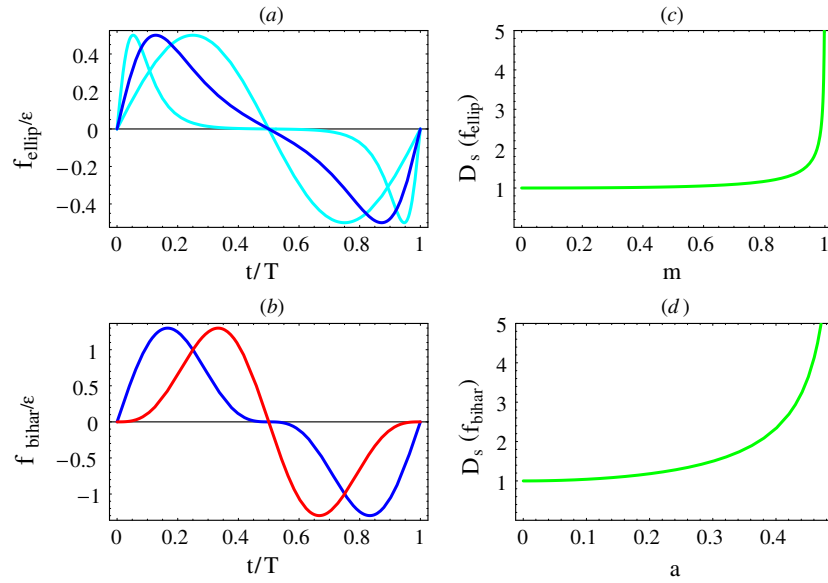


Figure 1. (a) Elliptic force $f_{\text{ellip}}(t) = \epsilon f(t; T, m, \theta) \equiv \epsilon sn(\Omega t + \Theta; m)cn(\Omega t + \Theta; m)$ versus t/T and three shape parameter values, $m = 0$ (light blue (light grey) line), $m = 0.983417$ (dark blue (dark grey) line, optimal waveform), $m = 1 - 10^{-6}$ (light blue (light grey) line), showing an increasing symmetry-breaking sequence as the pulse narrows, i.e., as $m \rightarrow 1$. (b) Optimal biharmonic force generating directed transport in one direction ($f_{\text{bihar}}^+(t)/\epsilon = \sin(2\pi t/T) + \frac{1}{2} \sin(4\pi t/T)$, dark blue (dark grey) line) and the opposite direction ($f_{\text{bihar}}^-(t)/\epsilon = \sin(2\pi t/T) - \frac{1}{2} \sin(4\pi t/T) \equiv -f_{\text{bihar}}^+(t + T/2)/\epsilon$, red (mid grey) line). (c) Measure of the DSB (equation (1)) for the elliptic force in (a), $D_s(f_{\text{ellip}}) = E(m)K^{-1}(m)(1-m)^{-1/2}$ versus m . One sees a sharp increase as $m \rightarrow 1$. (d) Measure of the DSB (equation (1)) for the biharmonic force $f_{\text{bihar}}(t)/\epsilon = \cos(2\pi t/T) + a \cos(4\pi t/T + \varphi_{\text{eff}})$, $D_s(f_{\text{bihar}})$ versus a for $\varphi_{\text{eff}} = \pi/2$. One sees a sharp increase as $a \rightarrow 1/2$, which is similar to that found in (c) for the elliptic function.

(This figure is in colour only in the electronic version)

Table 1. Optimal values of the relative amplitude ϵ_2/ϵ_1 and the effective phase $\varphi_{\text{eff}} \equiv \varphi_2 - 2\varphi_1$ obtained by computing the measure (1) of DSB for the three symmetries of the biharmonic force (second column), change rules of the effective phase under time transformations (third and fourth columns) and general scaling laws in leading order for averaged velocities and currents. Note the coherence of the results in the second and fifth columns, which were obtained using independent methods (recall that, without loss of generality, $\epsilon_1 + \epsilon_2 = 1$ so that $\epsilon_1^2 \epsilon_2 = (1 - \epsilon_2)^2 \epsilon_2$, which is a function having a single maximum at $\epsilon_2 = 1/3$, and hence $\epsilon_1 = 2/3$, $\epsilon_2/\epsilon_1 = 1/2$).

har ₁ , har ₂	$D_{s,+,-}$	$t \rightarrow t + t_0$	$t \rightarrow -t + t_0$	$\langle V \rangle$
cos, cos	$\frac{\epsilon_2}{\epsilon_1} = 1/2, \varphi_{\text{eff}} = \{\frac{\pi}{2}, \frac{3\pi}{2}\}$	$\tilde{\varphi}_{\text{eff}} = \varphi_{\text{eff}}$	$\tilde{\varphi}_{\text{eff}} = -\varphi_{\text{eff}}$	$\sim \epsilon_1^2 \epsilon_2 \sin \varphi_{\text{eff}}$
sin, sin	$\frac{\epsilon_2}{\epsilon_1} = 1/2, \varphi_{\text{eff}} = \{0, \pi\}$	$\tilde{\varphi}_{\text{eff}} = \varphi_{\text{eff}}$	$\tilde{\varphi}_{\text{eff}} = -\varphi_{\text{eff}} \pm \pi$	$\sim \epsilon_1^2 \epsilon_2 \cos \varphi_{\text{eff}}$
sin, cos	$\frac{\epsilon_2}{\epsilon_1} = 1/2, \varphi_{\text{eff}} = \{\frac{\pi}{2}, \frac{3\pi}{2}\}$	$\tilde{\varphi}_{\text{eff}} = \varphi_{\text{eff}}$	$\tilde{\varphi}_{\text{eff}} = -\varphi_{\text{eff}}$	$\sim \epsilon_1^2 \epsilon_2 \sin \varphi_{\text{eff}}$
cos, sin	$\frac{\epsilon_2}{\epsilon_1} = 1/2, \varphi_{\text{eff}} = \{0, \pi\}$	$\tilde{\varphi}_{\text{eff}} = \varphi_{\text{eff}}$	$\tilde{\varphi}_{\text{eff}} = -\varphi_{\text{eff}} \pm \pi$	$\sim \epsilon_1^2 \epsilon_2 \cos \varphi_{\text{eff}}$

DSB. This indicates that (1) provides a *bona fide* measure of the DSB. Remarkably, such optimal parameter values correspond to a *single* optimal waveform for the four versions of the biharmonic force (see figure 1(b)). The DSB mechanism implies that such a waveform is quite general, i.e., it corresponds to a force waveform which optimally enhances the ratchet effect in general systems. Consider now the case of the elliptic force

$$f_{\text{ellip}}(t) = \epsilon f(t; T, m, \theta) \equiv \epsilon sn(\Omega t + \Theta; m)cn(\Omega t + \Theta; m),$$

where $cn(\cdot; m)$ and $sn(\cdot; m)$ are Jacobian elliptic functions [27] of parameter m , $\Omega \equiv 2K(m)/T$, $\Theta \equiv K(m)\theta/\pi$, $K(m)$ is the complete elliptic integral of the first kind [27], T is the period of the force and θ is the (normalized) initial phase ($\theta \in [0, 2\pi]$). Fixing ϵ , T and θ , the force waveform changes as the shape parameter m varies from 0 to 1 (see figure 1(a)). In this case, equation (1) yields

$$D_s(f_{\text{ellip}}) = \frac{E(m)}{K(m)\sqrt{1-m}},$$

where $E(m)$ is the complete elliptic integral of the second kind [27] (see figure 1(c)). Physically, the motivation for this choice is that $f_{\text{ellip}}(t; T, m = 0, \theta) = \epsilon \sin(2\pi t/T + \theta)/2$ and that $f_{\text{ellip}}(t; T, m = 1, \theta)$ vanishes, i.e., in these two limits directed transport is not possible, while it is expected for $0 < m < 1$. Thus, one may expect in general the average of any relevant observable $\langle V \rangle$ to exhibit an extremum at a certain critical value $m = m_c$ as the shape parameter m is varied, the remaining parameters being held constant. The DSB mechanism implies that such a value m_c is quite general, i.e., it corresponds to a particular force waveform which optimally enhances the ratchet effect in general systems. *Generality* requires that such an optimal waveform should be closely related to that deduced for the case of a biharmonic force, in the sense of its Fourier series. Indeed, using

$$f_{\text{ellip}}(t; T, m, \theta)/\epsilon = \sum_{n=1}^{\infty} a_n(m) \sin \left[n \left(\frac{2\pi t}{T} + \theta \right) \right],$$

$$a_n(m) \equiv \frac{n\pi^2}{mK^2(m)} \operatorname{sech} \left[\frac{n\pi K(1-m)}{K(m)} \right]$$

one could expect the critical value m_c to be near $m = 0.983417$ since

$$\frac{f_{\text{ellip}}(t; T, m = 0.983417, \theta)}{\epsilon a_1(m = 0.983417)} = \sin \left(\frac{2\pi t}{T} + \theta \right) + \frac{1}{2} \sin \left(\frac{4\pi t}{T} + 2\theta \right) + 0.178592 \sin \left(\frac{6\pi t}{T} + 3\theta \right) + \dots,$$

i.e., the optimal values ($\epsilon_2/\epsilon_1 = 1/2$, $\varphi_{\text{eff}} = 0$) for the biharmonic approximation of the elliptic function are recovered at $m = 0.983417$ (cf table 1, second column, and compare figures 1(a) and (b)). Numerical studies of diverse systems [28] confirmed the accuracy of the critical value $m_c = 0.983\dots$, i.e., the effectiveness of the optimal waveform. Similarly, from the Fourier series of a sawtooth-wave force

$$f_{\text{sawtooth}}(t, T) = 2\epsilon \left[\sin \left(\frac{2\pi t}{T} \right) - \frac{1}{2} \sin \left(\frac{4\pi t}{T} \right) + \frac{1}{3} \sin \left(\frac{6\pi t}{T} \right) - \dots \right],$$

one recovers the optimal values ($\epsilon_2/\epsilon_1 = 1/2$, $\varphi_{\text{eff}} = \pi$) for its biharmonic approximation (cf table 1, second column), which explains the great effectiveness of this waveform in controlling directed transport of magnetic flux quanta [23].

Next, one exploits the aforementioned generality expected from the DSB mechanism to deduce the dependence of $\langle V \rangle$ on the symmetry-breaking parameters ($\epsilon_1, \epsilon_2, \varphi_1, \varphi_2$) of the biharmonic force $f_{h_1, h_2}(t)$ in leading order for the usual case [5–22, 24, 25] of small amplitudes ($1/\epsilon_{1,2} \rightarrow \infty$) when directed transport effectively occurs (note that, depending upon the particular system under study, the ratchet effect can yield steady transport or induced dipoles [22]). For the sake of clarity, first consider the case where the violation of the time-reversal symmetry of the system's dynamic equations can be absorbed in the temporal force because dissipation is negligible and the Lagrangian (Hamiltonian) of the system does not contain any additional term explicitly breaking the time-reversal symmetry. This means that the breaking

of the time-reversal symmetry implies the breaking of the force's symmetry $f(-t) = f(t)$. According to the above arguments, one generally expects $\langle V \rangle \sim s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2)$, where it is assumed without loss of generality that the function s is k -times piecewise continuously differentiable. From MacLaurin's series, one has

$$s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} c_{k,n}(\varphi_1, \varphi_2) \epsilon_1^k \epsilon_2^n$$

with $c_{k,0} = c_{0,n} = 0$ since the shift symmetry is never broken in the case of a single harmonic function. The transformation $\epsilon_i \rightarrow -\epsilon_i$, $i = 1, 2$, implies $f_{h1,h2}(t) \rightarrow -f_{h1,h2}(t)$, and hence $\langle V \rangle \rightarrow -\langle V \rangle$. This means that $s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) \rightarrow -s(-\epsilon_1, -\epsilon_2, \varphi_1, \varphi_2)$ and hence $k + n = 2m + 1$, $m = 1, 2, \dots$. Thus, one obtains

$$s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = c_{1,2}(\varphi_1, \varphi_2) \epsilon_1 \epsilon_2^2 + c_{2,1}(\varphi_1, \varphi_2) \epsilon_1^2 \epsilon_2 + O(\epsilon_1^2 \epsilon_2^3, \epsilon_1^3 \epsilon_2^2, \epsilon_1^4 \epsilon_2, \epsilon_1^4 \epsilon_2^4), \quad (2)$$

for $\epsilon_{1,2}$ sufficiently small. Since the ratchet effect does not depend on the time origin, $\langle V \rangle$ must remain invariant under the transformation $t \rightarrow t + t_0$, $\forall t_0$. This transformation yields $f_{h1,h2}(t) \rightarrow \epsilon_1 \text{har}_1(\omega t + \tilde{\varphi}_1) + \epsilon_2 \text{har}_2(2\omega t + \tilde{\varphi}_2)$, with the fundamental property $\tilde{\varphi}_2 - 2\tilde{\varphi}_1 = \varphi_2 - 2\varphi_1$, i.e., the effective phase φ_{eff} remains invariant under time translation (see table 1, third column), and hence

$$\langle V \rangle \sim c_{1,2}(\varphi_{\text{eff}}) \epsilon_1 \epsilon_2^2 + c_{2,1}(\varphi_{\text{eff}}) \epsilon_1^2 \epsilon_2. \quad (3)$$

The transformation $\varphi_i \rightarrow \varphi_i - \pi$, $i = 1, 2$, implies $f_{h1,h2}(t) \rightarrow -f_{h1,h2}(t)$, and hence $\langle V \rangle \rightarrow -\langle V \rangle$ whereby $c_{1,2}(\varphi_{\text{eff}}) = -c_{1,2}(\varphi_{\text{eff}} + \pi)$, $c_{2,1}(\varphi_{\text{eff}}) = -c_{2,1}(\varphi_{\text{eff}} + \pi)$, while the transformation $\epsilon_1 \rightarrow -\epsilon_1$, $\varphi_1 \rightarrow \varphi_1 - \pi$ maintains $\langle V \rangle$ invariant, and hence $c_{1,2}(\varphi_{\text{eff}}) = -c_{1,2}(\varphi_{\text{eff}} + 2\pi)$, $c_{2,1}(\varphi_{\text{eff}}) = c_{2,1}(\varphi_{\text{eff}} + 2\pi)$. The comparison of these four relationships for the functions of the effective phase implies that $c_{2,1}(\varphi_{\text{eff}})$ is a 2π -periodic function while $c_{1,2}(\varphi_{\text{eff}}) \equiv 0$. Thus, equation (3) reduces to

$$\langle V \rangle \sim (1/\epsilon_1)^{-2} (1/\epsilon_2)^{-1} c_{2,1}(\varphi_{\text{eff}}), \quad (4)$$

where a *power law* for the dependence on the amplitudes is now explicit. In this regard, it is worth noting the great similarity between the present theory and the highly optimized tolerance (HOT) theory [29] where a power law is generated by the actions of an external agent aiming to optimize the behaviour of a system. However, we have seen above that generality comes from *criticality* in the present theory, while for HOT systems the details matter. To obtain an explicit expression for the function $c(\varphi_{\text{eff}})$ it is useful to consider the general transformation $t \rightarrow -t + t_0$, $\forall t_0$. This transformation yields $f_{h1,h2}(t) \rightarrow \epsilon_1 \text{har}_1(\omega t + \tilde{\varphi}_1) + \epsilon_2 \text{har}_2(2\omega t + \tilde{\varphi}_2)$ where the effective phase is no longer strictly invariant but changes according to table 1 (fourth column). Since the transformation $t \rightarrow -t$ implies $\langle V \rangle \rightarrow -\langle V \rangle$ when the time-reversal symmetry is unbroken, the change rules of the effective phase imply that the function $c_{2,1}(\varphi_{\text{eff}})$ has necessarily a definite parity (cf equation (4) and table 1, fourth column). Taking into account this property and given that $c_{2,1}(\varphi_{\text{eff}})$ is assumed to be k -times piecewise continuously differentiable, its Fourier series [30] can be approximated to leading non-trivial order by a single harmonic function according to table 1 (fifth column). One sees that the general scaling laws in table 1 (fifth column) yield $\langle V \rangle = 0$ when and only when both the shift symmetry and the time-reversal symmetry of the system's dynamic equations (i.e., the force's symmetry $f(-t) = f(t)$ in the present case) are unbroken, while they yield a maximum value of $\langle V \rangle$ when and only when maximal *effective* (critical) symmetry breaking occurs in the sense of the measure (1), as predicted from the DSB mechanism. Also, that the harmonic functions appearing in the scaling laws are independent of har_1 is a consequence of the invariance of $\langle V \rangle$ under time translation. As expected, one finds that such scaling laws are quite general,

i.e., they confirm and explain previous results for a great diversity of systems [7, 14, 15, 22, 25] subjected to a biharmonic force $f_{h1,h2}(t)$.

I now discuss how the aforementioned scaling laws change when the violation of the time-reversal symmetry of the system's dynamic equations cannot be absorbed in the temporal force. This is the case when dissipation [5, 6, 8–13, 16–21, 24] is not negligible. The approximate conservation of the effective phase in the sense of the change rules in table 1 (fourth column) has been demonstrated above, and hence that φ_{eff} is the proper argument of the function $c_{2,1}(\varphi_1, \varphi_2)$, when the violation of the time-reversal symmetry only occurs in the temporal force. Therefore, that the violation of such a symmetry is also due to the presence of dissipation means that φ_{eff} can no longer be an argument of the function $c_{2,1}(\varphi_1, \varphi_2)$ but one has $\varphi_{\text{eff}} + \varphi_{\text{diss}}$ instead, where φ_{diss} is hereafter referred to as the *dissipation phase*. Note that the additive character of the dissipation phase is a consequence of the DSB mechanism. Thus, the dissipation phase quantifies the degree of breaking of the time-reversal symmetry generated by dissipation. Also, the DSB mechanism implies the general properties: $\varphi_{\text{diss}}(\beta = 0) = 0$ and that $\varphi_{\text{diss}}(\beta)$ is a monotonically increasing function of β , with β being the effective dissipation parameter. For the values of φ_{eff} yielding $\langle V \rangle = 0$ in the absence of dissipation, i.e., those values for which the temporal force does not break the time-reversal symmetry of a non-dissipative system, one obtains that the maximum absolute value (i.e., 1) of the harmonic functions appearing in the scaling laws is reached at $\varphi_{\text{diss}} = \pm\pi/2$, and hence we have the additional general property $\max_{\beta} \varphi_{\text{diss}}(\beta) = \pi/2$ (cf table 1, fifth column). However, the function $\varphi_{\text{diss}}(\beta)$ generally depends upon additional parameters, such as the period and diverse system-dependent parameters. Of course, it is generally expected that the function $\langle V \rangle / \text{har}(\varphi_{\text{eff}} + \varphi_{\text{diss}})$ should exhibit monotonically decreasing behaviour as a function of β , where *har* is the corresponding harmonic function in table 1 (fifth column) in each case. When dissipative forces dominate inertia (the so-called overdamped regime [1, 4]), the breaking of the time-reversal symmetry implies the breaking of the force's symmetry $f(-t) = -f(t)$ and the dissipative phase reaches its limiting values $\varphi_{\text{diss}} = \pm\pi/2$. Since the optimal values of the relative amplitude ϵ_2/ϵ_1 and the effective phase φ_{eff} are just the same for the three symmetries of the biharmonic force, this means that the general scaling laws corresponding to the overdamped regime are those given in table 1 (fifth column) but with *sin* instead of *cos*, and vice versa, in each case. One finds that these predictions are in perfect agreement with published results for a great diversity of systems [4–6, 8–13, 16–21, 24] subjected to a biharmonic force $f_{h1,h2}(t)$. Since dissipative forces and randomly fluctuating forces (noise) have the same microscopic origin, the effectiveness of temporal forces at generating directed transport induced by the ratchet effect is expected to be robust against moderate presence of noise.

In summary, the present work establishes that to optimally enhance directed transport by symmetry breaking of temporal forces there exists a particular force waveform which allows to deduce simple scaling laws from a quantitative interpretation of Curie's principle. This conjecture explains in a general setting all previously published results for a great diversity of systems [5–25] and provides for the first time a quantitative criterion to optimize any application of the ratchet effect induced by symmetry breaking of temporal forces. The scope and validity of such a conjecture should be matters of further research, and the author would welcome insights from his colleagues.

Acknowledgments

The author warmly thanks N R Quintero for fruitful discussions. This study was supported by the Spanish MEC through FIS2004-02475 project.

References

- [1] Feynman R P, Leighton R B and Sands M 1966 *The Feynman Lectures on Physics* vol 1 (Reading, WA: Addison-Wesley) chapter 46
- [2] Hänggi P and Bartussek R 1996 *Lecture Notes in Physics* vol 476 ed J Parisi *et al* (Berlin: Springer) p 294
- [3] Linke H (ed) 2002 *Ratchets and Brownian Motors: Basics, Experiments and Applications* (special issue) *Appl. Phys. A* **75** 2
- [4] Reimann P 2002 *Phys. Rep.* **361** 57
- [5] Seeger K and Maurer W 1978 *Solid State Commun.* **27** 603
- [6] Ajdari A, Mukamel D, Peliti L and Prost J 1994 *J. Phys. I: France* **4** 1551
- [7] Dupont E, Corkum P B, Liu H C, Buchanan M and Wasilewski Z R 1995 *Phys. Rev. Lett.* **74** 3596
- [8] Alekseev K N, Erementchouk M V and Kusmartsev F V 1999 *Europhys. Lett.* **47** 595
- [9] Flach S, Yevtushenko O and Zolotaryuk Y 2000 *Phys. Rev. Lett.* **84** 2358
- [10] Yevtushenko O, Flach S, Zolotaryuk Y and Ovchinnikov A A 2001 *Europhys. Lett.* **54** 141
- [11] Salerno M and Zolotaryuk Y 2002 *Phys. Rev. E* **65** 56603
- [12] Flach S, Zolotaryuk Y, Miroshnichenko A E and Fistul M V 2002 *Phys. Rev. Lett.* **88** 184101
- [13] Engel A, Müller H W, Reimann P and Jung A 2003 *Phys. Rev. Lett.* **91** 60602
- [14] Schiavoni M, Sánchez-Palencia L, Renzoni F and Grynberg G 2003 *Phys. Rev. Lett.* **90** 94101
- [15] Fistul M V, Miroshnichenko A E and Flach S 2003 *Phys. Rev. B* **68** 153107
- [16] Lehmann J, Kohler S, Hänggi P and Nitzan A 2003 *J. Chem. Phys.* **118** 3283
- [17] Morales-Molina L, Quintero N R, Mertens F G and Sánchez A 2003 *Phys. Rev. Lett.* **91** 234102
- [18] Ustinov A V, Coqui C, Kemp A, Zolotaryuk Y and Salerno M 2004 *Phys. Rev. Lett.* **93** 87001
- [19] Gommers R, Bergamini S and Renzoni F 2005 *Phys. Rev. Lett.* **95** 73003
- [20] Hänggi P, Marchesoni F and Nori F 2005 *Ann. Phys., Leipz.* **14** 51
- [21] Xie X, Dai J and Zhang X-C 2006 *Phys. Rev. Lett.* **96** 75005
- [22] Franco I and Brumer P 2006 *Phys. Rev. Lett.* **97** 40402
- [23] Cole D *et al* 2006 *Nature Mat.* **5** 305
- [24] Zolotaryuk Y and Salerno M 2006 *Phys. Rev. E* **73** 66621
- [25] Denisov S, Morales-Molina L and Flach S 2006 *Preprint cond-mat/0607558*
Denisov S, Morales-Molina L, Flach S and Hänggi P 2007 *Preprint quant-ph/0703169*
- [26] Curie P 1894 *J. Phys. (Paris)* **3** 393
- [27] Zwillinger D 2003 *Standard Mathematical Tables and Formulae* (London: Chapman and Hall/CRC Press)
- [28] Chacón R and Quintero N R 2007 *BioSystems* **88** 308
- [29] Newman M 2000 *Nature* **405** 412
Carlson J M and Doyle J 2000 *Phys. Rev. Lett.* **84** 2529
- [30] Katznelson Y 1976 *An Introduction to Harmonic Analysis* (New York: Dover)